

Parameterization and Reconstruction of Quasi Static Universe

Jie Liu^{*} and Yun-Song Piao[†]

*College of Physical Sciences, Graduate University of
Chinese Academy of Sciences, Beijing 100049, China*

Abstract

We study a possibility of the fate of universe, in which there is neither the rip singularity, which results in the disintegration of bound systems, nor the endless expansion, instead the universe will be quasi static. We discuss the parameterization of the corresponding evolution and the reconstruction of the scalar field model. We find, with the parameterization consistent with the current observation, that the current universe might arrive at a quasi static phase after less than 20Gyr.

^{*}Electronic address: liujie10b@mails.gucas.ac.cn

[†]Electronic address: yspiao@gucas.ac.cn

I. INTRODUCTION

The observations imply that the current universe is undergoing an acceleration [1], which is driven by the dark energy, or leaded by the modification of gravity on large scale. The simplest candidate of dark energy is the cosmological constant, but it suffers from the cosmic coincidence problem. Thus the dynamical dark energy might be favored, there have been some candidates of scalar field model such as [2],[3],[4],[5],[6], see e.g.[7],[8] for reviews.

The fate of universe is determined by the nature of dark energy. The universe driven by the phantom will evolve to a singularity, in which the energy density become infinite at finite time, which is called the big rip, see also Ref.[9] for other future singularities. How avoiding these singularities is still an interesting issue, e.g.[10] and the little rip scenario in which a rip singularity can not occur at finite time [11],[12].

In principle, due to the difference nature of dark energy, the universe may have different date. Here, we will study a possibility of the fate of universe, in which the universe will be quasi static some time after the current time. In this fate of universe, there is neither the disintegration of bound systems, i.e. the rip singularity, nor the going of stars and galaxies, i.e. the endless expansion, e.g. in dS universe or the universe dominated by matter, instead the scale factor of universe will be nearly constant.

The outline of paper is as follows. We will firstly study how to parameterize the evolution of a quasi static universe in section II, and then will bring some specific cases in section III. The reconstruction of the scalar field model is given in section IV. The discussion is given in section V.

II. HOW TO PARAMETERIZE THE EVOLUTION OF A QUASI STATIC UNIVERSE

The intending universe is static, in which a tends to constant, requires that

$$H \longrightarrow 0, \text{ and } \int^t H dt \longrightarrow 0 \tag{1}$$

for $t \rightarrow \infty$ should be satisfied. The corresponding evolutions can be parameterized as

$$H \sim \frac{1}{t_0} \left(\frac{t_0}{t} \right)^b, \quad (2)$$

$$\text{or } \text{Exp} \left(- \left(\frac{t}{t_0} \right)^k \right), \quad (3)$$

where $b > 1$ and $k > 0$ are constant. That with $0 < b \leq 1$, e.g. $H \sim 1/t$, is not suitable, since $\int H dt \rightarrow 0$ is diverged. Here, the behavior of $H \rightarrow 0$ is power law or is exponential. However, of course, the behavior of H could be also double exponential

$$H \sim \text{Exp} \left(-e^{\left(\frac{t}{t_0} \right)} \right), \quad (4)$$

$$\text{or } \text{higher exponential.} \quad (5)$$

Here, for (2), $a = e^{\int H dt}$ is given by

$$a \sim a_{static} \text{Exp} \left(\left(\frac{t_0}{t} \right)^{b-1} \right). \quad (6)$$

Thus in the regime $t \gg t_0$, the universe is asymptotically static, where a_{static} is the static value of a .

In this parameterization, the universe asymptotically arrives at the static phase. However, it may be nearly static some time after t_0 . We define the time when e.g. $a \sim 0.99a_{static}$ as the static time t_{static} , which means at the time that the universe is quasi static. With (6), t_{static} is given by

$$t_{static} = \frac{1}{(\ln 0.99)^{1/(b-1)}} t_0. \quad (7)$$

We see that the larger b is, the earlier t_{static} is, since the Hubble parameter decays faster for larger b .

However, the expansion of the current universe is accelerated, thus a consistent parameterization should not only overlap the above parameterization for $t \gg t_0$, but also give the current acceleration around $t \simeq t_0$.

III. APPLICATION TO SPECIFIC CASES

We will bring some specific parameterizations in this section, and discuss the details of the models and restrict the parameters in parameterizations with current observation.

A. The universe with parameterization (i)

$$H(t) = \frac{A}{t_0} \left(\frac{t}{t_0} \right)^{k-1} \text{Exp} \left(-B \left(\frac{t}{t_0} \right)^k \right) \quad (8)$$

The model have three parameters A , B , and k , and obviously, $A, B > 0$ and $k > 1$. With the Hubble parameter (8), a is given by

$$a(t) = \text{Exp} \left(\int H(t) dt \right) = a_{static} e^{-\frac{A}{kB} \text{Exp} \left[-B \left(\frac{t}{t_0} \right)^k \right]}. \quad (9)$$

The relationships between the three parameters and the current a_0 and H_0 are

$$a_0 = a_{static} e^{-\frac{A}{kB} \text{Exp}(-B)}, \quad (10)$$

and

$$H_0 = \frac{A e^{-B}}{t_0}. \quad (11)$$

The universe consists of the dark matter and the dark energy. We have, for FRW space-time,

$$\rho_{DE} = \frac{3}{\kappa^2} H^2 - \rho_{DM}, \quad (12)$$

$$p_{DE} = -\frac{2}{\kappa^2} \dot{H} - \frac{3}{\kappa^2} H^2, \quad (13)$$

where $\kappa^2 = 8\pi G$. The density of dark matter is given as

$$\rho_{DM} = \rho_{DM0} \text{Exp} \left[-\frac{3A}{kB} \left(e^{-B} - e^{-B \left(\frac{t}{t_0} \right)^k} \right) \right]. \quad (14)$$

Thus the required ρ_{DE} and p_{DE} are only determined by the parameterization of H or a . The equation of state parameter of the dark energy is $\omega_{DE} = p_{DE}/\rho_{DE}$. In infinite latetime, both H and \dot{H} tend to 0, we have $\rho_{DE} \simeq -\rho_{DM}$ and $p_{DE} \simeq 0$. Thus at infinite latetime, it is required that $\rho_{DE} < 0$, which just sets off the positive density of dark matter, and $\omega_{DE} \simeq 0$ is the same as that of dark matter, which insures that the decaying of their energy density with time are same. Here, the dark energy may be the field or fluid, which satisfies (12) and (13). The derivation of the Hubble parameter is

$$\dot{H} = \frac{A}{t_0^2} \left(\frac{t}{t_0} \right)^{k-2} \left[k - 1 - Bk \left(\frac{t}{t_0} \right)^k \right] e^{-B \left(\frac{t}{t_0} \right)^k}. \quad (15)$$

The expansion of universe at t_0 is accelerated. The quasi static phase implies that the universe must begin deceleration at the time $t_{tr} > t_0$, in which $\ddot{a} = 0$. With $\ddot{a}/a = \dot{H} + H^2$ and (15), we get

$$k - 1 - kB \left(\frac{t_{tr}}{t_0} \right)^k + A \left(\frac{t_{tr}}{t_0} \right)^k \text{Exp} \left[-B \left(\frac{t_{tr}}{t_0} \right)^k \right] = 0. \quad (16)$$

Thus we have the condition

$$\frac{1}{1-B} > k > \frac{Ae^{-B} - 1}{B - 1}. \quad (17)$$

With (9), the universe is quasi static at t_{static}

$$t_{static} = \left(\frac{1}{B} \ln \frac{-A}{kB \ln 0.99} \right)^{1/k} t_0. \quad (18)$$

We restrict the space of the parameters A and B with observation in [13], in which $-1.033 \leq \omega_\phi \leq -0.927$, and the condition (17) for different k , see Fig.1. We find the range of the parameter space decrease with the raising k .

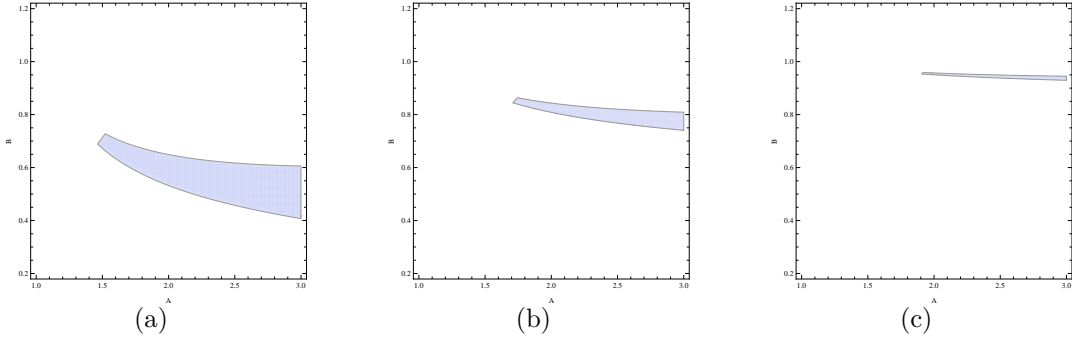


FIG. 1: The observation constrains to the (A, B) parameters space for the different k . (a): $k=1.5$, (b): $k=3$, (c): $k=10$.

In Fig.2, we plot the evolution of relevant quantities numerically, in units of ρ_{c0}^{-1} , correspondingly the space of the parameters in Fig.1, where $\rho_{c0} = 3H_0/\kappa^2$ is the present critical density of the universe. Here we take $\Omega_{DM0} = 0.27$. We find that the value of the parameter A has a range which is agreement with the current observations, i.e. $\Omega_{DE} \sim 3\Omega_{DM}$.

In the Table I, we list t_{static} and t_{tr} by choosing three sets of the special parameter A and B from different k . We see that the larger k is, the earlier both t_{tr} and t_{static} are, since the Hubble parameter decays faster for larger k .

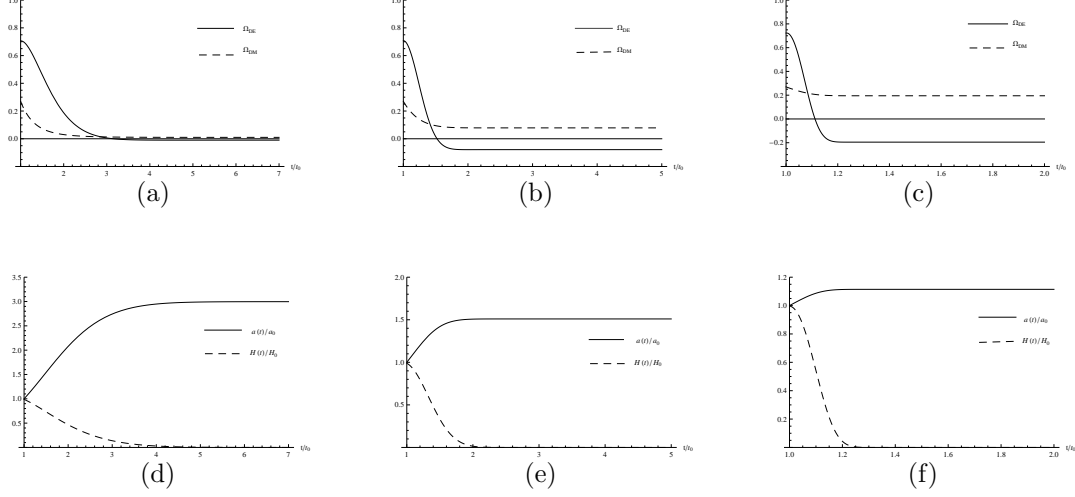


FIG. 2: The upper panels are the graphs of the evolutions of Ω_{DM} and Ω_{DE} ; the lower panels are the graphs of the evolution of $a(t)$ and $H(t)$ by normalized. (a) and (d):the used parameter values are $k=1.5$, $(A, B)=(1.8, 0.6)$; (b) and (e):the used parameter values are $k=3$, $(A, B)=(2.2, 0.8)$; (c) and (f):the used parameter values are $k=10$, $(A, B)=(2.5, 0.92)$.

k	(A, B)	t_{tr}/t_0	$t_{tr} - t_0$	t_{static}/t_0	$t_{static} - t_0$
1.5	1.8, 0.6	1.468	64.116Gyr	4.270	447.990Gyr
3	2.2, 0.8	1.079	10.823Gyr	1.780	106.860Gyr
10	2.5, 0.92	1.008	1.096Gyr	1.136	18.632Gyr

TABLE I: The transition time t_{tr} and the static time t_{static} for the different parameter spaces.

B. The universe with parameterization (ii)

$$H(t) = \frac{A}{t_0} \left(\frac{t_0}{t} \right)^b \left[1 - B \left(\frac{t_0}{t} \right) \right] \quad (19)$$

The model have three parameters A , B , and b , and obviously, $A, B > 0$ and $b > 2$. The discussion is similarly to the model in subsection A. We will not repeat it, and only plot the evolution of relevant quantities numerically in Fig.3 for a set of parameters. The transition time is $t_{tr} = 1.003t_0$, while the static time is $t_{static} = 7.804t_0$.

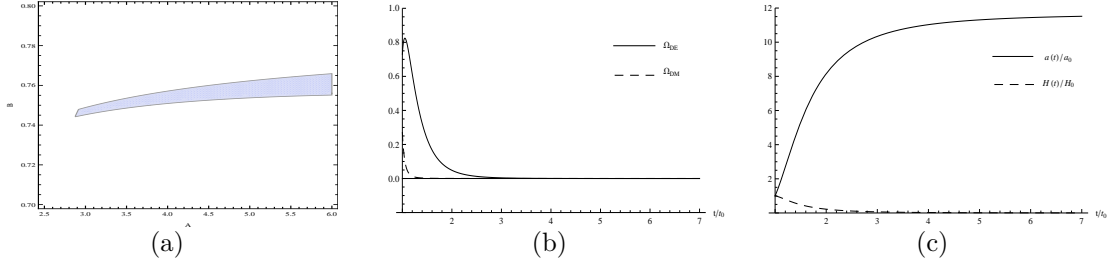


FIG. 3: The left panel is the graph of the observation constraints to the (A, B) parameters space for $b = 3.5$; The middle panel is the graph of the evolutions of Ω_{DM} and Ω_{DE} ; The right panel is the graph of the evolutions of $a(t)$ and $H(t)$. The used parameter values are $b = 3.5$, $(A, B) = (4, 0.75)$

IV. THE RECONSTRUCTION OF SCALAR FIELD MODEL

We will reconstruct the scalar field model with the parameterization (8). The reconstruction of scalar field dark energy models has been studied in lots of references, e.g.[14],[15],[16],[17],[18].

The Lagrangian density of scalar field is given by $P(X, \phi)$, where $X = -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi / 2$. Thus the energy density $\rho_{DE} = 2XP_X - P$ is derived. In a spatially flat FRW universe, the equations for the dark matter and the scalar field are

$$\frac{3}{\kappa^2} H^2 = \rho_{DM} + 2XP_X - P, \quad (20)$$

$$\frac{2}{\kappa^2} \dot{H} = -\rho_{DM} - 2XP_X, \quad (21)$$

where $\rho_{DM} = \rho_{DM0}(\frac{a_0}{a})^3$. Thus we have

$$P = -\frac{2}{\kappa^2} \dot{H} - \frac{3}{\kappa^2} H^2, \quad (22)$$

$$\dot{\phi}^2 P_X = -\frac{2}{\kappa^2} \dot{H} - \rho_{DM}. \quad (23)$$

In principle, after specifying $P(X, \phi)$, we can have the evolution of $\dot{\phi}$ and P with the time. Thus the potential function in $P(X, \phi)$ can be reconstructed.

Here, we apply a generalized ghost condensate Lagrangian, e.g.[15], in which $P = -X + h(\phi)X^2$. Thus with Eqs.(22) and (23), we have

$$\dot{\phi}^2 = \frac{6}{\kappa^2} \dot{H} + \frac{12}{\kappa^2} H^2 - \rho_{DM}, \quad (24)$$

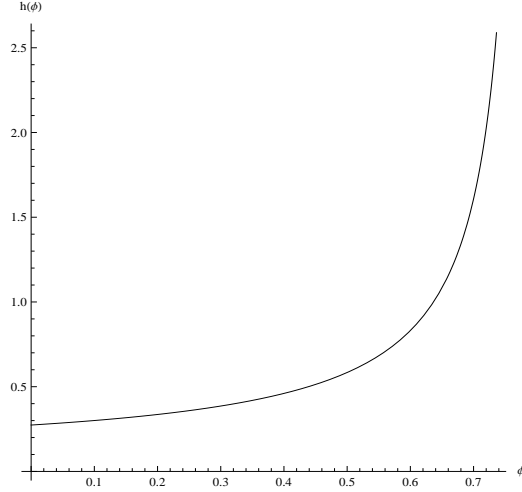


FIG. 4: The reconstruction for the function of $h(\phi)$ according to the model A with $k = 1.5$ and $(A, B) = (1.8, 0.6)$.

$$h(\phi) = \frac{\frac{4}{\kappa^2}\dot{H} + \frac{12}{\kappa^2}H^2 - 2\rho_{DM}}{(\frac{6}{\kappa^2}\dot{H} + \frac{12}{\kappa^2}H^2 - \rho_{DM})^2}. \quad (25)$$

With (24) and (25), one can reconstruct the function of $h(\phi)$ for the generalized ghost condensate model in the quasi static universe. In Fig.4, we plot the reconstruction for $h(\phi)$ according to with the parameterization (8) numerically, in units of ρ_{co}^{-1} , and the special parameters are $k = 1.5$ and $(A, B) = (1.8, 0.6)$. Where, we have fixed the field amplitude at the present epoch to be zero: $\phi(t_0) = 0$.

V. DISCUSSION

The fate of the universe is an interesting issue. We study a possibility of the fate of universe, in which the universe is quasi static some time after t_0 . This model may has a region of parameter space in which it resemble Λ CDM, which thus is not conflicted with the current observation.

In section III.A, we study the parameterization (8). When $t > t_0$, $H(t)$ is rapidly decreased, thus we will go in the static universe shortly after t_0 . We find that the current universe might arrive at the static phase after about 18.632 Gyr. For the universe, it is a very brief spell.

In section III.B, we study the parameterization (19). Here, since $H(t)$ is decreased slower than that in section III.A, we will take longer time to go in the static universe. With specific

parameterization consistent with the current observation, we find that the current universe will arrive at the static phase after $t_{static} = 7.804t_0$, about 932.148 Gyr.

Here, we show the possibility of an alternative fate of universe, in which the universe is quasi static some time after t_0 . In this fate of universe, there is neither the disaggregation of bound systems, nor the going of stars and galaxies, the universe will have a quiet “afternoon”.

The reconstruction of the scalar field models of quasi static universe is significant. We have discussed a case. However, we have neglected the coupling between the scalar field and the dark matter. The case including the coupling is also interesting, which will be a substantial work.

Acknowledgments

We thank Hong Li, Taotao Qiu and Yuanzhong Zhang for helpful discussions. This work is supported in part by NSFC under Grant No:11075205, in part by the Scientific Research Fund of GUCAS(NO:055101BM03), in part by National Basic Research Program of China, No:2010CB832804

-
- [1] A.G. Riess *et al.*, Astron. J. **116**, 1009 (1998); S. Perlmutter *et al.*, Astrophys. J. **517**, 565 (1999).
 - [2] C. Wetterich, Nucl. Phys. B302, 668 (1988); B. Ratra and P. J. E. Peebles, Phys. Rev. D37, 3406 (1988); E. J. Copeland, A. R. Liddle, and D. Wands, Ann. N. Y. Acad. Sci. 688, 647 (1993); P. G. Ferreira and M. Joyce, Phys. Rev. Lett. 79, 4740 (1997); Phys. Rev. D 58, 023503 (1998); I. Zlatev, L. M. Wang and P. J. Steinhardt, Phys. Rev. Lett. 82, 896 (1999); P. J. Steinhardt, L. M. Wang and I. Zlatev, Phys. Rev. D 59, 123504 (1999).
 - [3] G. W. Gibbons, Phys. Lett. B 537, 1 (2002); T. Padmanabhan, Phys. Rev. D 66, 021301 (2002); J. S. Bagla, H. K. Jassal and T. Padmanabhan, Phys. Rev. D 67, 063504 (2003).
 - [4] R. R. Caldwell, Phys. Lett. B 545, 23 (2002); R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phys. Rev. Lett. 91, 071301 (2003); P. Singh, M. Sami and N. Dadhich, Phys. Rev. D 68, 023522 (2003); Z.K. Guo, Y.S. Piao, Y.Z. Zhang, Phys. Lett. B594, 247 (2004).
 - [5] B. Feng, X.L. Wang, X.M. Zhang, Phys. Lett. B607, 35 (2005); Z. K. Guo, Y. S. Piao, X. M. Zhang and Y. Z. Zhang, Phys. Lett. B 608, 177 (2005); B. Feng, M.Z. Li, Y.S. Piao, X.M. Zhang, Phys. Lett. B634, 101 (2006); X. Zhang, Commun.Theor.Phys.44, 762 (2005); Phys.

- Rev.D74, 103505, (2006); Y. F. Cai, E. N. Saridakis, M. R. Setare and J. Q. Xia, Phys. Rept. 493, 1 (2010).
- [6] H. Wei, R.G. Cai, D.F. Zeng, Class.Quant.Grav.22, 3189 (2005); H.Wei, R.G. Cai, Phys.Lett. B634, 9 (2006).
- [7] E.J. Copeland, M. Sami, S. Tsujikawa, Int.J.Mod.Phys.D15, 1753 (2006).
- [8] K. Bamba, S. Capozziello, S. Nojiri, S. D. Odintsov, arXiv:1205.3421.
- [9] S. Nojiri, S.D. Odintsov, S. Tsujikawa, Phys. Rev. **D71**, 063004 (2005); Y. Shtanov, V. Sahni, Class. Quant. Grav. **19**, L101 (2002); J.D. Barrow, Class. Quant. Grav. **21**, L79 (2004); Class. Quant. Grav. **21**, 5619 (2004); H. Stefancic, Phys. Rev. **D71**, 084024 (2005); I.H. Brevik, O. Gorbunova, Gen. Rel. Grav. **37**, 2039 (2005); M.P. Dabrowski, Phys. Lett. **B625**, 184 (2005); P.H. Frampton, K.J. Ludwick, R.J. Scherrer, Phys. Rev. **D85**, 083001 (2012).
- [10] S. Nojiri and S. D. Odintsov, Phys. Lett. B 686, 44 (2010); E. Elizalde, S. Nojiri and S. D. Odintsov, Phys. Rev. D 70, 043539 (2004); K. Bamba, S. Nojiri and S. D. Odintsov, JCAP 0810, 045 (2008); S. Nojiri and S. D. Odintsov, Phys.Rept.505 59, (2011).
- [11] P.H. Frampton, K.J. Ludwick and R.J. Scherrer, Phys. Rev. D 84 (2011) 063003; P. H. Frampton, K. J. Ludwick, S. Nojiri, S. D. Odintsov and R. J. Scherrer, Phys. Lett. B 708 (2012) 204.
- [12] A.V. Astashenok, S. Nojiri, S.D. Odintsov and A. V. Yurov, Phys.Lett. B709, 396 (2012); A.V. Astashenok, S. Nojiri, S.D. Odintsov and R.J. Scherrer, arXiv:1203.1976; Y. Ito, S. Nojiri and S.D. Odintsov, arXiv:1111.5389; K. Bamba, R. Myrzakulov, S. Nojiri and S. D. Odintsov, arXiv:1202.4057; E. Elizalde, A.N. Makarenko, S. Nojiri, V.V. Obukhov and S.D. Odintsov, arXiv:1206.2702; A.V. Astashenok, E. Elizalde, S.D. Odintsov and A.V. Yurov, arXiv: 1206.2192; L. N. Granda and E. Loaiza, arXiv:1111.2454; P. Xi, X.H. Zhai and X.Z. Li, Phys. Lett. B 706 (2012) 482; A. N. Makarenko, V.V. Obukhov and I. V. Kirnos, arXiv:1201.4742; Z.G. Liu and Y.S. Piao, Phys. Lett. B 713 (2012) 53; P. C. Stavrinos and S.I. Vacaru, arXiv:1206.3998.
- [13] E. Komatsu, K. M. Smith, J. Dunkley, C. L. Bennett, B. Gold, G. Hinshaw, N. Jarosik, D. Larson, M. R. Nolta, L. Page, et al., The Astrophysical Journal Supplement Series 192, 57 (2010).
- [14] T. D. Saini, S. Raychaudhury, V. Sahni and A. A. Starobinsky, Phys. Rev. Lett. 85, 1162 (2000); A. A. Starobinsky, JETP Lett. 68, 757 (1998); D. Huterer and M. S. Turner, Phys.

- Rev. D 60, 081301 (1999); T. Nakamura and T. Chiba, Mon. Not. R. Astron. Soc. 306, 696 (1999).
- [15] S. Tsujikawa, Phys. Rev. D 72, 083512 (2005).
- [16] S. Capozziello, V.F. Cardone, E. Elizalde, S.Nojiri and S. D.Odintsov, Phys. Rev. D 73, 043512 (2006); S. Nojiri and S. D. Odintsov, Phys. Rev. D 72, 023003 (2005); S. Capozziello, S. Nojiri and S. D. Odintsov, Phys. Lett. B 632, 597 (2006).
- [17] Z. K. Guo, N. Ohta and Y. Z. Zhang, Phys. Rev. D 72, 023504 (2005); Z. K. Guo, N. Ohta and Y. Z. Zhang, Mod. Phys. Lett. A 22, 883 (2007).
- [18] X. Zhang, Phys. Lett. B 648, 1 (2007); J. Zhang, X. Zhang and H. Liu, Phys. Lett. B 651, 84 (2007).